Nonlocal supercurrent of quartets in a three-terminal Josephson junction

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Methods and Supplemental Information:
In this Supplementary Section we add details to the main text. We include a brief review of the theoretical background as well as the simulation method and results, as well as more information on the conductance and noise measurements.

S1 - Theoretical Model

A. General
Let us consider a normal region connected to three superconducting terminals. When terminals $S_{L,R}$ are biased respectively at voltages $V_{L,R}$ with respect to terminal $S_M$, a coherent stationary motion of Cooper pairs occurs when $nV_L+mV_R=0$, where $(n,m)$ are integers. This involves $n$ pairs crossing from $S_M$ to $S_L$ and $m$ pairs crossing from $S_M$ to $S_R$ in a single quantum process [1, 2]. This multi-pair process unveils a phase combination $\varphi_{n,m} = n\varphi_L + m\varphi_R - (n+m)\varphi_M$ which, owing to the Josephson relation, $\frac{d\varphi_i}{dt} = \frac{2eV_i}{h}$; $(i=L,R,M)$, is a constant of motion. The main anomaly reported in the experiment along the line $V_L+V_R=0$ corresponds to a quartet (a pair of pairs) crossing from $S_M$ towards $S_{L,R}$, revealing the stationary phase $\varphi_q = \varphi_{1,1} = \varphi_L + \varphi_R - 2\varphi_M$. Sextet lines are also visible, though fainter, where, $(n,m) = (1,2)$ or $(2,1)$. These DC modes manifest static phase
coherence despite the non-equilibrium conditions. Due to energy conservation, multi-pair processes are non-dissipative, contrary to the usual quasiparticle multiple Andreev reflections (MAR). Along the line $V_L=V_R=V$, theory predicts that the quartet current $I_q(\varphi_q,V)$ is odd in phase and even in voltage. $I_q$ is similar to a DC Josephson supercurrent but it depends on $V$ as a new control parameter. It involves equal and perfectly correlated currents flowing through $S_L$ and $S_R$.

Choosing $\varphi_q = \varphi_L + \varphi_R - 2\varphi_M$ and $\chi = \varphi_L - \varphi_R$, as canonical variables one may as a first step begin with the Andreev bound state (ABS) energies at equilibrium $E_{ABS}(\varphi_q,\chi)$ which can be computed in a suitable model. Subsequently, one can use a semiclassical approximation and average out the drifting phase $\chi(t)$. This can be formally done by expanding $E_{ABS}(\varphi_q,\chi)$ in Fourier series in both variables keeping only the zeroth order component in $\chi(t)$. This leads to an effective energy $E_{eff}(\varphi_q)$, which is a function of $\varphi_q$ only. Then the average quartet current is found to be $I_{\text{quartet}}^{\text{SC}} = \frac{2e}{\hbar} \frac{dE_{eff}}{d\varphi_q}$. This rough procedure reduces a set of two-dimensional ABS, valid at equilibrium, to a set of one-dimensional effective ABS. Yet, it neglects the quantum nature of the non-equilibrium processes, which take place as multiple Andreev reflections at the junction interface of all three superconductors. In the limit where the Josephson junction frequency $\omega_0 = \frac{2eV}{\hbar}$ is much smaller than the separation between the effective ABS, one obtains Landau-Zener transitions between the latter. None equilibrium Green’s function calculations confirm this picture (see below and Figure S1a) and demonstrate that those transitions indeed induce a strong quartet noise.

**B. Results from non-equilibrium Green’s function theory**

The picture above is semi-phenomenological and a full non-equilibrium theory of transport is necessary. Such a theory is indeed available along the line $V_L=-V_R=V$; it involves the calculation of the Keldysh Green’s function matrix $G(E,n)$, where $E$ is the energy and $n$ the index of the harmonics of the Josephson frequency $\omega_0 = \frac{2eV}{\hbar}$ [2, 3]. Voltages down to 0.1$\Delta$ can be reached with about 100 harmonics. Mapping the full $(V_L,V_R)$ plane is out of reach, as independent Josephson frequencies $\omega_L, \omega_R$ would require much too large matrices. Results concerning a single dot model are found in Ref. [3]. The model used to describe the present experiment is similar but
instead involves two single-level quantum dots $D_L$ and $D_R$ with energy levels $\varepsilon_L$ and $\varepsilon_R$ coupled to the terminals by couplings (broadening in the normal state) $\Gamma_L$, $\Gamma_M$ (for dot $D_L$), and $\Gamma_R$, $\Gamma_M$ (for dot $D_R$). For the purpose of interpreting the experiment, $\varepsilon_L$ and $\varepsilon_R$ are taken to be zero (resonant dots). Interactions are neglected owing to the large contact transparency. Figure S1a shows the quartet current flowing in terminal M and the cross-correlation noise $S_{LR}$. The $\Gamma$’s are taken as $\Gamma_L = \Gamma_R = 1.5\Delta$ and a smaller $\Gamma_M = 0.3\Delta$, owing to the finite width of the central superconducting finger that limits the crossed Andreev reflection.

Panels S1a, A-B show the quartet current and the crossed noise as a function of the quartet phase, fixing $eV = 0.15\Delta$ and taking into account a very small inelastic broadening $\eta = 10^{-6}$ [in units of $\Delta$] in the superconductors. A very strong resonance appears as marked dips at specific values of $\phi_q$, that can be interpreted as resonant Landau-Zener transitions between two symmetrical ABS formed at zero voltage, triggered by the Josephson frequency $\omega_0$. This indeed resembles the effect of microwave irradiation on a quantum point contact [4]. Spectacularly, the cross correlation noise exhibits sharp peaks at the same phase values as the current dips (Fig S1a, Panel B). These peaks can be very high, signaling “trains” of quartets, in a way similar to the thermal noise due to transitions between a single junction ABS [5, 6]. Fig S1a, Panel C & D shows a broadening and an amplitude decrease in the current and the noise anomalies when increasing the inelastic parameter, where $\eta = 10^{-3}$. Panel S1a, E shows the variation with $V$ of the value of the cross correlation noise, calculated along the line $(V, -V)$ of the $G_L(V_L, V_R)$ map by taking into account thermal fluctuations (see Section C). First, one finds that the noise is positive. Second, its behavior is not monotonous, the first maximum being indeed due to the above Landau-Zener resonance. The maximum noise is much larger than $\frac{e^2\Delta}{\hbar}$, indicating large bursts of quartets emitted within the Landau-Zener resonances. Those trends are also found in the experiment, where a non-monotonous variation of the maximum noise is obtained as well (Figure 4c, main text). No quantitative fit is attempted here, because the details of the current and noise variations with phase and voltage are very sensitive to the location of the resonances. In particular, i) the non-monotonous variation with $V$, with huge oscillations, and ii) the anharmonic phase variation, with dips reflecting Landau-Zener transitions, are characteristic of such resonances and point towards
the phase coherence of the quartet dynamics. Here, the parameters of the model are chosen to illustrate the main trends in a somewhat dramatic case. We also emphasize the extreme sensitivity of the quartet noise to the inelastic time, a parameter unknown in the experiment.

C. Phase diffusion model close to the quartet line

Here we present a semi-phenomenological picture which is capable of describing transport in the vicinity of the quartet line \((V_L, V_R)\), where no full microscopic solution is available anymore. In a voltage-biased junction, the Josephson supercurrent is probed indirectly through the shape of the conductance anomaly manifesting a rounded Josephson plateau in the \(V(I)\) characteristics. Its double-well shape can be described by an overdamped RSJ model [7-8]. The same is true here for the conductance anomaly, as a function of two voltages \(V_L, V_R\). Transport by a quartet supercurrent is witnessed by a rounded plateau, centered on the quartet line.

To describe the anomaly in the \((V_L, V_R)\) plane, one resorts to a semi-phenomenological description, based on a generalization of the RSJ model (Fig. S1b)). It consists of two Josephson junctions in parallel, with one common contact, and two parallel channels for currents flowing through them: i) the quartet channel, with \(-\) by construction \(-\) equal currents in both branches; ii) the MAR current channels, that are different in each branch, due to the possibly different transparencies: they can be further decomposed into a phase-dependent component and a phase-independent one. Those encompass both local MAR (occurring within one junction only) and nonlocal ones (coupling all three contacts). Here one neglects thermal quasiparticle transport. The currents thus read:

\[
I_L = I_Q(\varphi_Q) + I_{L}^{ph-MAR}(\varphi_Q) + I_{L}^{MAR}
\]

\[
I_R = I_Q(\varphi_Q) + I_{R}^{ph-MAR}(\varphi_Q) + I_{R}^{MAR}
\]

(1)

where \(I_Q(\varphi_Q)\) depends only on \(V\) but the MAR components depend both on \(V_L\) and \(V_R\). From now on we neglect the phase-MAR terms, which in the relevant voltage regime are of high order and just correct the main MAR contributions.

Let us consider the vicinity of the « diagonal » \(V_L = -V_R\) line. One defines \(v_L\) and \(v_R\) such as \(V_L = V + v_L, V_R = -V + v_R\), and set \(v = v_L + v_R\) as the deviation from the diagonal, assuming
\[ |v_L|, |v_R| \ll |V| \]. The quartet anomaly develops on a small scale (corresponding to the small quartet critical current) while MAR evolve smoothly in the \((V_L, V_R)\) plane. One can thus linearize the MAR contributions and write:

\[
I^\text{MAR}_L (V_L, V_R) \approx I^\text{MAR}_L (V, -V) + g_{LL} v_L + g_{LR} v_R
\]

\[
I^\text{MAR}_R (V_L, V_R) \approx I^\text{MAR}_R (V, -V) + g_{RL} v_L + g_{RR} v_R
\]

(2)

To go further, let us remark that the variable \(\phi_Q = \phi_L + \phi_R\) is slow while the other canonical variable \(\chi = \phi_L - \phi_R\) is fast, e.g.

\[
\dot{\phi}_Q = \frac{2e}{\hbar} v, \ \dot{\chi} = \frac{4e}{\hbar} V
\]

(3)

Using \(\dot{\phi}_{LR} = \frac{2e}{\hbar} V_{LR}\), and expressing \(v_L, v_R\) in terms of \(\dot{\phi}_Q\) and \(\dot{\chi}\) yields:

\[
v_L = -V + \frac{\hbar}{4e} (\phi_Q + \dot{\chi}), \ \ n_{LR} = V + \frac{\hbar}{4e} (\phi_Q - \dot{\chi})
\]

(4)

and equations (1,2) become

\[
I_L = I_Q(\phi_Q) + I^\text{MAR}_L (V, -V) + (g_{LR} - g_{LL})V + \frac{\hbar}{4e} [(g_{LL} + g_{LR}) \phi_Q + (g_{LL} - g_{LR}) \dot{\chi}]
\]

\[
I_R = I_Q(\phi_Q) + I^\text{MAR}_R (V, -V) + (g_{RR} - g_{RL})V + \frac{\hbar}{4e} [(g_{RL} + g_{RR}) \phi_Q + (g_{RL} - g_{RR}) \dot{\chi}]
\]
One can perform a coarse-graining of Eqs. (5), averaging out the fast oscillations at the Josephson frequency $\frac{4e}{h} V$. This amounts to drop all terms in $\dot{\chi}$ and yields two RSJ-like equations for the same phase $\phi_Q$ (assuming a sinusoidal quartet current):

$$\dot{\phi}_Q \approx \frac{4e}{h g_L} \left( I'_L - I_Q^0 \sin \phi_Q \right) \approx \frac{4e}{h g_R} \left( I'_R - I_Q^0 \sin \phi_Q \right)$$

(6)

with

$$g_L = g_{LL} + g_{LR}, \quad g_R = g_{RL} + g_{RR}$$

(7)

containing both local and nonlocal MAR, and

$$I'_L = I_L - I_L^{MAR} (V, -V) - (g_{LR} - g_{LL}) V$$

(8)

$$I'_R = I_R - I_R^{MAR} (V, -V) - (g_{RR} - g_{RL}) V$$

(9)

The constraint contained in Eq. (6) enforces the link between currents $I_L$ and $I_R$. The phase dynamics amounts to a thermal diffusion at temperature $T$ in either of the (equivalent) potentials $U_{L,R}(\phi_Q) = I_Q^0 (1 - \cos \phi_Q) - I_{L,R}^0 \phi_Q$. This was solved by Ambegaokar and Halperin for the classic Josephson junction problem [8]. Here, the calculation yields the average phase velocity (thus voltage deviation with respect to the quartet line) as a function of the currents $< I_L >$ or $< I_R >$, averaged on the phase distribution resulting from phase diffusion:
\[ v = \frac{\hbar}{2e} < \phi_Q > = \frac{4iQ^0}{\gamma g_{LR}} \frac{e^{\gamma g_{LR}}}{e^{\gamma g_{LR}}} \left[ \int_0^{2\pi} e^{-\frac{\gamma g_{LR} Q}{2}} I_0 \left( \gamma \sin \frac{\phi_Q}{2} \right) d\phi_Q \right]^{-1} \]  

(11)

with \( \alpha_{LR} = \frac{<I_{LR}^0>}{iQ^0} \) and \( \gamma = \frac{\hbar iQ^0}{e k_B T} \) and \( I_0 \) is the modified Bessel function of the first kind. We do not need to explicitly calculate this dependence which is well tabulated and parametrized by the critical current \( I_Q^0 \) (given by a Green's function calculation, or better fitted to the experiment), by the temperature and by the quasiparticle resistances associated to MAR processes. From Eqs.(6-9), one sees by subtraction of \( I_L^0 \) and \( I_R^0 \) that the quartet contribution disappears, which leaves :

\[ <I_L^0> - <I_R^0> = \frac{\hbar}{4e} (g_L - g_R) <\phi_Q> = \frac{1}{2} (g_L - g_R) v \]  

(12)

In the experiment, the conductances are measured as \( G_L = \frac{\partial I_L}{\partial V_M} \) and \( G_R = \frac{\partial I_R}{\partial V_M} \), where \( V_M \) is a small ac voltage applied in the middle contact. This amounts to shift the voltages \( V_{LR} \) as \( V_L - V_M, V_R - V_M. \) It results in

\[ G_L = 2 \frac{\partial I_L}{\partial v}, \quad G_R = 2 \frac{\partial I_R}{\partial v} \]  

(13)

From Eq. (12), one obtains

\[ G_L - G_R = g_L - g_R \]  

(14)

with \( g_L - g_R = g_{LL} - g_{RR} \). This simple result is fully consistent with the experiment in which the traces of \( G_L \) and \( G_R \) at fixed \( V_R \) are parallel, their offset reflecting the different MAR conductances in both branches. This is a striking consequence of the fact that the quartet current is common to both
branches and is at the origin of the anomaly. The difference between $G_L$ and $G_R$, of order $\frac{2e^2}{h}$, is a signature of the asymmetry in the MAR process.

The above model maps the – nonequilibrium - quartet anomaly onto that of a – close to equilibrium - standard Josephson junction, treated in the RSJ model. Solution of Eq. (11) yields the usual S-shape of the $I(V)$ characteristics – here $l_0(v)$ or equivalently $l_0(V_l)$ at fixed $V_R$ - thus the conductance anomaly represented in Fig. S1c. It displays a maximum flanked by two flat minima, and its width measures the order of magnitude of an equivalent quartet “Josephson energy” $E_0$ and of the quartet “critical current”. With $E_q \approx \frac{\hbar V_c}{2e} = 2\mu eV$, the critical quartet supercurrent is about 0.6nA.

This argument confirms that the conductance anomaly across the quartet line underlies the quartet phase, and allows to evaluate a typical quartet energy to be about 60-100mK. This model also allows calculating the thermally averaged value of the crossed noise at the center of the anomaly ($v = 0$). It is plotted in Figure S1a, Panel E and can be much larger than $\frac{e^2\Delta}{\hbar}$. Yet, this model does not allow to fully calculating the crossed correlation noise anomaly across the quartet $(V, -V)$ line, owing to the strong non adiabatic character of the quartet noise which dramatically depends on Landau-Zener transitions.

**D. Nonlocal multiple Andreev reflections vs quartets.**

We now discuss the zero-energy nonlocal MAR process which might compete with the quartet mechanism along the line $(V, -V)$. In the main text, we explain several observations that distinguish between the two effects. Perhaps the most important one is the sign of the crossed noise measured by correlating the current fluctuations on the left and right terminals. While the measured signal is positive, the crossed noise expected from the zero-energy nonlocal MAR process is negative. This can be first understood by an intuitive argument: in a zero-energy MAR –particles are transported between terminals at different voltages with the help of energy-conserving Cooper pair transitions. Such a fermionic dissipative transport is expected to result in anti-bunching, e.g. negative noise correlations between terminals at different voltages, thus negative CC between L, R terminals. This is indeed confirmed by a full Keldysh calculation, made with a single level dot model, taking into account all conserving MAR processes together with quartets [3]. Figure 5 of Ref. 3 shows an essential result: it compares the quartet current $I_c$ in terminal C (the setup is symmetrical), the
quasiparticle current (Ia-Ib) and the noise correlation CC, for different values of the voltage V. For large voltages the MAR current dominates the quartet current and the CC is indeed weak and negative (red and green curves in panel a, b, c of Figure 5, obtained for a resonant junction, as in the experiment). On the contrary, for small voltages, the MAR current is smaller than the quartet current and the CC is strong and positive (black and light blue curves). This phenomenon is generic and observed for a more realistic two-level dot model as well.

**S2 - Measurement Setup**

The experimental setup is shown in Fig. S2a. Resonance frequencies of the two LC circuits were matched in order to enable the cross-correlation measurements at ~705KHz.

**A. Differential conductance measurements:**

As described in the main text, differential conductance was measured by applying an input ac signal of 0.8\(\mu\)Vrms at 705 KHz to the center contact, \(S_M\), while measuring the differential voltages, \(V_L\) and \(V_R\), on the left and right contacts, \(S_L\) and \(S_R\), respectively. The 500\(\Omega\) load resistors were chosen to be significantly lower than the typical values of the sample resistance so that they serve as effective drains pulling most of the current to the ground. We then define:
\[G_L = dI_L/dV_M, \quad G_R = dI_R/dV_M,\]
where \(I_L = V_L/500\Omega\) and \(I_R = V_R/500\Omega\). Figure S2b presents a color plot of \(G_L\) and \(G_R\) as a function of the applied biases \(V_L\) and \(V_R\) in device \(dI\).

**B. Cross-correlation of current fluctuations measurements:**

In the cross-correlation of current fluctuations measurement no AC signal is applied. DC bias voltages, however, produce current fluctuations, \(dI_L\) and \(dI_R\) (ac component at relatively low frequencies ~ 705kHz). We are interested in the cross correlation of the current fluctuations \(<dI_LdI_R>\). The current fluctuations introduce voltage fluctuations \(dV_L = dI_L \times 500\Omega\) and \(dV_R = dI_R \times 500\Omega\) at the inputs of a home-made, cold (1K) amplifier (the gains of which were measured in advance to be \(g_L = 6.12\) and \(g_R = 5.77\)). Another amplification stage was used at the output of the dilution fridge using NF amplifiers each with a gain of 200. Both signals are multiplied and amplified by a home-made cross correlator with a central frequency of 730KHz,
resolution band width of RBW=100KHz and gain of \( g_{cc}^2 = 10^7 \). Finally, the cross correlator signal undergoes an RC filter. The CC can be estimated by:

\[
CC_{tot} = \langle (dI_L \times 500 \times g_L \times g_{NF} \times g_{cc}) \times (dI_R \times 500 \times g_R \times g_{NF} \times g_{cc}) \rangle \times RBW
\]

\[
= \langle dI_L dI_R \rangle \times [500^2 \times g_L \times g_R \times g_{NF}^2 \times g_{cc}^2 \times RBW]
\]

\[
= \langle dI_L dI_R \rangle \times \alpha
\]

However, parasitic effects such as RF picked up by both output lines, cross talk coming from capacitance between the output lines etc., add an independent “background” cross correlation, \( CC_0 = \langle dI_L dI_R \rangle \times \alpha + CC_0 \).

Since the load resistor was chosen to be very small (500Ω) relative to the sample resistance, the voltage signal is very small, relative to the background cross correlation. Hence, the background must be calibrated and subtracted as explained in the next section.

**S3 – Cross-correlation calibration**

To calibrate the background, before each measurement of \( V_L \) where we scan the cross correlation (as we move through the quartet line), we perform the same measurement at a high magnetic field of \( B=200\text{mT} \) (above the critical field of the SCs so that all contacts are in the normal state). At zero bias voltages, no current flows through the device and we expect the voltage fluctuations \( dV_L \) and \( dV_R \) to be uncorrelated. Hence, we take the cross correlation measured at this high magnetic field and at zero bias as our background cross correlation. An example of such cross correlation measurement is shown in Fig. S3.

**S4 – Negative cross-correlation on the complementary quartet lines**

As mentioned in the main text we expect to observe a positive cross-correlation of the current fluctuations, between the left and right terminals, along the quartet conductance line. As a sanity check we measured the CC along different processes where we expect to get negative cross correlation.

In Fig. S4A we illustrated a quartet process which is named complementary quartet process, which is merely a permutation of the terminals from the process described in the main text. In this process Cooper pairs from the left and center contact enter the right contact and in the process they are entangled between themselves. This process is thus called a complementary
quartet process. In Fig S4B we sketched the complementary quartet ABS which is the mechanism for the creation of a quartet in the right contact.

The results of this process are shown in Fig S4C. We placed the right terminal at a bias of \( V_R = -13(\mu V) \) and measured the differential conductance in the left and right contact (upper and middle panels) and the cross correlation of the current fluctuations in the left and right terminals. Concentrating on the blue shaded region (where the complementary quartet process occurs) a clear reduction in the cross correlation is observed – originating from a negative contribution of the process. Concentrating now on the red shaded area, which is the region where a trivial supercurrent flows from the left to the right terminal, once again a clear reduction of the cross correlation is observed.

Fig S4D shows a SEM image of a T-shape nanowire configuration showing similar results.

**S5 – Confirming the absence of a common element in device d2**

To demonstrate experimentally that there is no normal element which is common to the two junctions in device d2, we performed CC measurements in the normal state of the devices, at high magnetic field. A fixed voltage \( V_R = -14 \mu V \) was applied to the right contact, \( S_R \), and the CC of the current fluctuations on \( S_L \) and \( S_R \) was measured as a function of \( V_L \). As shown in Fig. S5A, in the presence of a common element, the current \( I_R \) going to \( S_R \), would be correlated to the current \( I_L \) coming from \( S_L \). Thus, a finite CC would be measured. In contrast, in the absence of such common element (or if this element is significantly smaller than the other two, \( R_C \ll R_L, R_R \)), the currents \( I_L \) and \( I_R \) are completely independent and the CC would vanish, as seen in Fig. S5B. The same argument evidently holds in the tunneling regime as well, where instead of three resistive elements, there are three tunnel junctions. If the probability, \( \Gamma_C \), of tunneling from the center of the junction to the central contact \( S_M \), is comparable to the other two tunneling probabilities, then the currents \( I_L \) and \( I_R \) would be correlated. However if \( \Gamma_C \ll \Gamma_L, \Gamma_R \), then the CC would vanish. The measurement result is shown in Fig. S5C. Indeed we measured zero CC which demonstrates the absence of the common element.
References


Fig. S1a. Two-dot model.

NEFG calculation of the quartet current $I_0$ (in the central terminal M) and of the current cross correlation (CC) $S_{LR}$, as a function of the quartet phase $\varphi_q$ (panels A-D) and of the voltage (panel E), with $eV = 0.15\Delta$, the inelastic parameter $\eta_x = 10^{-6}\Delta$ (panels A, B) and $\eta_x = 10^{-3}\Delta$ (panels C, D, E). Panel A shows two resonances corresponding to Landau Zener transitions between the adiabatic quartet states. At the same phase values, the CC displays sharp maxima with very high values. Both trends are still present with a larger inelastic parameter (panels C, D) but broader and reduced amplitude. Panel E shows the non-monotonous variation of the CC at the center of the quartet ($\nu$ is the deviation from the quartet line) line but sweeping the voltage $V$, showing that the resonances occur at certain $V$ values. Thermal fluctuations are considered with gamma $= 0.5$ (see text).
**Fig. S1b. Phenomenological quartet RSJ model**

The quartet mode and the quasiparticle processes (local and nonlocal MAR’s) are represented by a three-terminal RSJ model, with a common “quartet” Josephson junction and two non-symmetrical dissipative branches.

**Fig. S1c. Differential conductance of the quartet**

Differential conductance as a function of voltage $v$, taken from the center of the quartet line, in units of the maximum quartet current, the normal state conductance being taken to one. It is calculated from the effective phase diffusion model, plugging in it the results of the Green’s function calculation for $IQ(\phi_Q)$.
Fig. S2a. Device and Measurement Setup.

SEM image: D1 configuration. Three superconducting contacts are placed on a single nanowire. The central contact is made much narrower than the coherence length of the superconductor so that crossed Andreev reflection is allowed. Our measurement setup. The differential conductance and the cross correlation of current fluctuations were measured using this setup as described above. The device configuration is schematically illustrated at the top right hand corner.
Fig. S2b. Differential conductance measurements results

A) $G_L = \frac{dI_L}{dV_m}$ vs. $V_R$ and $V_L$ measured in device $d1$. B) $G_R = \frac{dI_R}{dV_m}$ vs. $V_R$ and $V_L$ measured in device $d1$. 
Fig. S3. Cross correlation calibration.

Cross correlation as a function of $V_L$, measured at $B=200$ mT, well above the critical magnetic field of aluminum. At zero bias, there is no current and therefore we expect zero cross correlation. It can be seen that even at a finite bias of $50\mu$V the cross correlation is essentially the same as at zero bias (within an uncertainty that is significantly smaller than the cross correlation measured at $B=0$ on the quartet resonance). This is due to the fact that in device $d1$, the central contact disconnects the two sides of the device, or in other words most of the current from the left/right contact flows to the central contact rather than to the other side. Upper panel: All measured noise scans, lower panel: averaged.
Fig. S4. Complementary quartet lines.

A) Schematic illustration of the ‘right’ (right contact) complementary quartet line. B) Schematic illustration of the 3-terminal complementary quartet ABS. C) The upper and middle panels show the differential conductance in the left and right terminals. The lower panel displays the cross correlation of the current fluctuation, showing a negative peak in the location of the complementary quartet. D) SEM image: T-shape nanowire device.
Fig. S5. Absence of a common element. A) Common element, $R_c$, is comparable in size to $R_L$ and $R_R$: cross-correlation of current fluctuation is expected to have a positive sign. B) No common element, $R_c$ is comparable in size to $R_L$ and $R_R$: cross-correlation goes to zero. C) Zero CC as a function of $V_L$ proving scenario B is our case.